

Closing Tues: HW 9.7(2), 9.8, 9.9

Exam 1 is Thur, Jan. 25th covers 9.3 - 9.9.

9.9 Applications (continued)

Recall from last class (and Math 111):

If $p(x)$ = selling price per item (demand)

$AC(x)$ = average cost per item

then $TR(x) = p(x) \cdot x$

$TC(x) = AC(x) \cdot x.$

Entry Task (directly from HW 9.9/5)

The price of a certain product is \$400.

The cost per unit of producing the product is $130 + 0.5x$ dollars/item.

Give the functions $TR(x)$, $TC(x)$, $MR(x)$ and $MC(x)$.

$$TR(x) = p \cdot x = 400x$$

$$\Rightarrow \boxed{TR(x) = 400x}$$
$$MR(x) = TR'(x) = 400$$

$$TC(x) = AC(x) \cdot x = (130 + 0.5x)x$$

$$\Rightarrow \boxed{TC(x) = 130x + 0.5x^2}$$
$$MC(x) = TC'(x) = 130 + x$$

Recall from Math 111:

Profit and marginal profit are given by

$$P(x) = TR(x) - TC(x)$$

$$MP(x) = MR(x) - MC(x)$$

When profit is maximized

$$MR(x) = MC(x)$$

Specifically, where it switches from $MR > MC$ to $MR < MC$.

$$P(x) = TR(x) - TC(x)$$

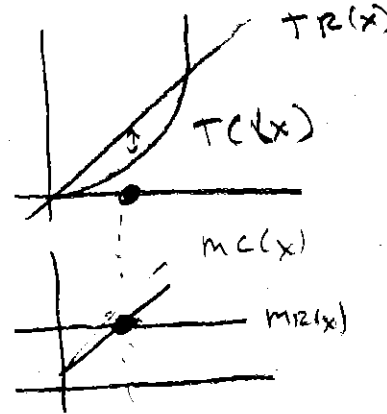
$$= (400x) - (130x + 0.5x^2)$$

$$= 400x - 130x - 0.5x^2$$

$$P(x) = 270x - 0.5x^2$$

$$MP(x) = P'(x) = 270 - x$$

Continuing example from the entry task:
How many units should the firm produce and sell to maximize its profits?



$$MR(x) = MC(x)$$

$$400 = 130 + x$$

$$\Rightarrow x = 270 \text{ items}$$

$$MP(x) = 0$$

$$\Rightarrow 270 - x = 0$$

$$\Rightarrow x = 270 \text{ items}$$

$$P(270) = 270(270) - 0.5(270)^2$$

occurs here

$$= 836,450 \leftarrow \text{MAX PROFIT}$$

Another example
(directly from an old midterm):

You sell items.

If q is in **hundred items**, then $TR(q)$ and $TC(q)$ in **hundred dollars** are given by

$$TR(q) = 30q$$

$$TC(q) = q^3 - 15q^2 + 78q + 10$$

a. Find marginal cost at 2 hundred items

$$MC(q) = TC'(q) = 3q^2 - 30q + 78$$

$$MC(2) = 3(2)^2 - 30(2) + 78$$
$$= \boxed{30} \quad \boxed{\$/item}$$

"The 201st item costs about \$30 to produce."

b. Find the longest interval over which marginal revenue exceeds marginal cost. WANT TO KNOW WHEN $MR(x) > MC(x)$

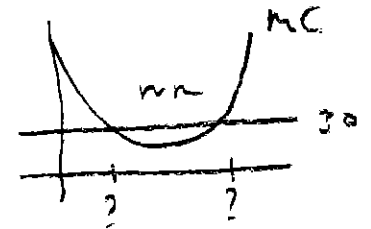
STEP 1 FIND $MR = MC$

$$30 = 3q^2 - 30q + 78$$

$$\Rightarrow 0 = 3q^2 - 30q + 48$$

$$0 = q^2 - 10q + 16 = (q - 2)(q - 8)$$

$$q = 2 \quad \text{or} \quad q = 8 \quad \left. \vphantom{q = 2} \right\} \text{OR USE QUAD. FORMULA TO GET THESE}$$



STEP 2 INTERPRET

MR is above MC between $q=2$ AND $q=8$

NOTE: PROFIT IS INCREASING FROM $q=2$ TO $q=8$, SO

MAX PROFIT OCCURS AT $q=8$

c. What is the maximum value of profit?

$$P(Q) = TR(Q) - TC(Q)$$

$$TR(Q) = 30(Q) = 240$$

$$TC(Q) = (Q)^3 - 15(Q)^2 + 76(Q) + 10$$
$$= 186$$

$$P(Q) = 240 - 186$$

$$= 54 \text{ hundred dollars}$$

$$(\$5,400)$$

Graphs and Derivatives

Example: Let $f(x) = 2x^2 - 3x$

Find $f'(x) = 4x - 3$

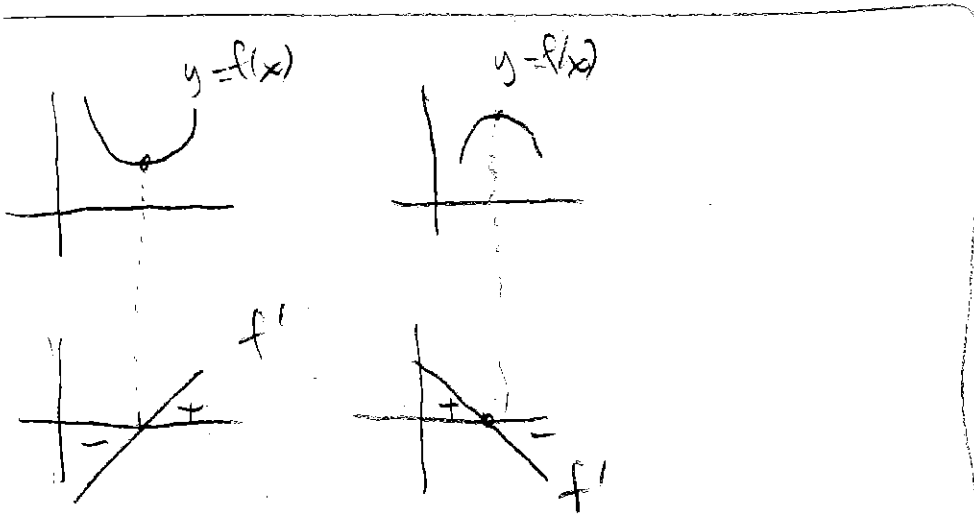
Note:

I $f'(x) = 0$ when

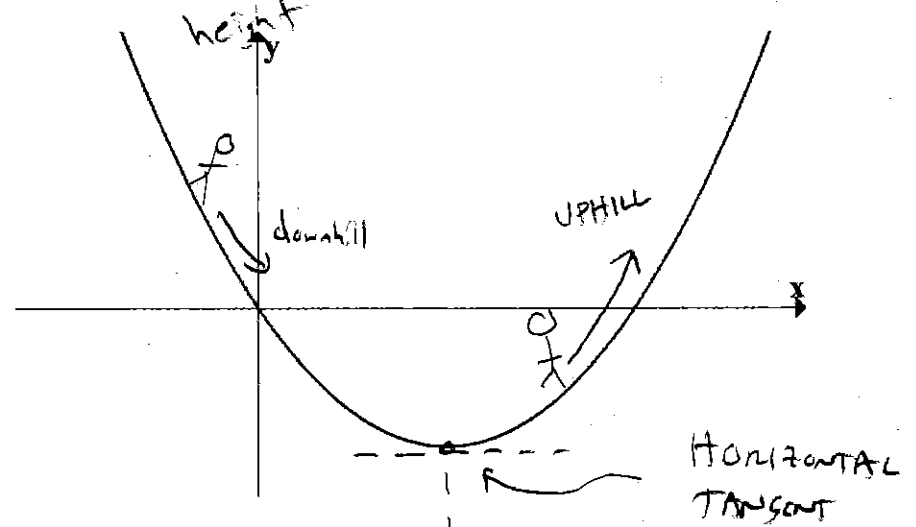
$$4x - 3 = 0 \iff x = \frac{3}{4}$$

II f' IS POSITIVE For ALL $x > \frac{3}{4}$

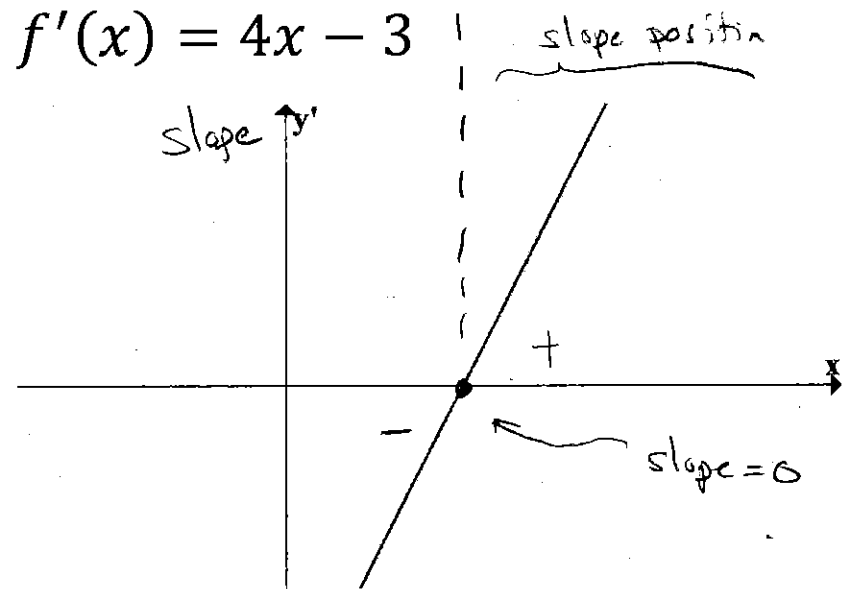
III f' IS NEGATIVE For ALL $x < \frac{3}{4}$



$$f(x) = 2x^2 - 3x$$



$$f'(x) = 4x - 3$$



Notes/Observations: Given $y = f(x)$.

- $y = f'(x)$ is a new function.
- $f(x)$ = “height of the graph at x ”
- $f'(x)$ = “slope of $f(x)$ at x ”
- $f'(x)$ is “instantaneous rate of change” (speedometer speed)
- The units of $f'(x)$ are $\frac{y\text{-units}}{x\text{-units}}$.

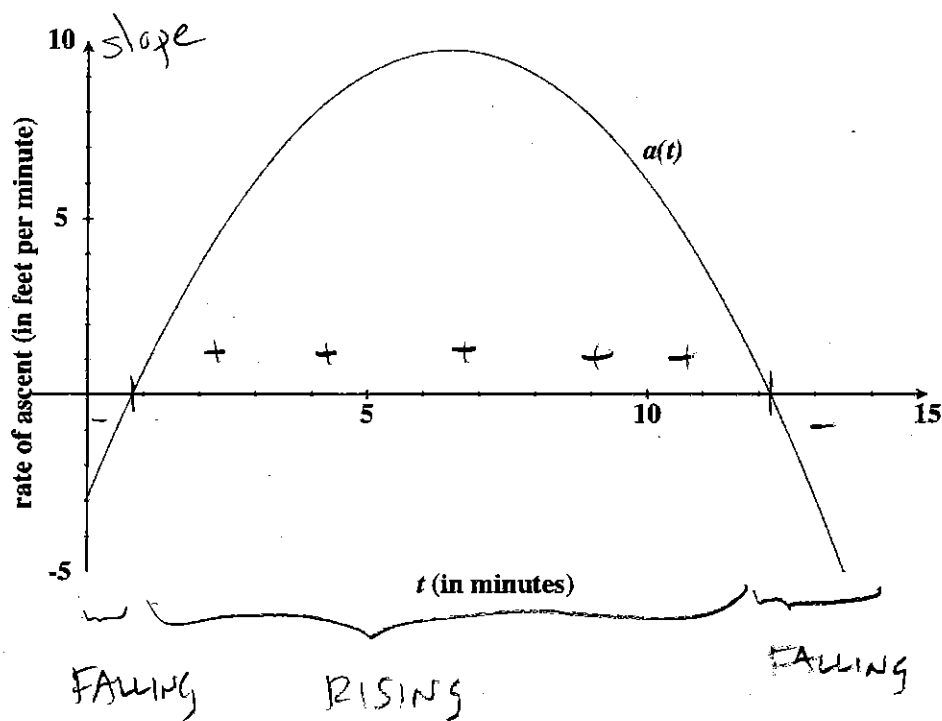
Fundamental to all applications:

$f(x)$	$f'(x)$
horiz. tangent	zero
increasing	positive
decreasing	negative

9.9 HW Problem 8:

Rate of ascent for a balloon (in feet per minute) is given by

$$a(t) = -0.3t^2 + 3.9t - 2.928$$



How will you answer these:

- (a) Find the longest interval over which Balloon A is rising.
- (d) Find the time at which the balloon is rising the fastest?

(a) NEED TO FIND WHEN

$a(t)$ IS POSITIVE.

STEP 1 FIND WHEN $a(t) = 0$

$$-0.3t^2 + 3.9t - 2.928 = 0$$

QUAD. FORMULA

STEP 2 INTERPRET

BETWEEN THESE

(d) WANT TO FIND HIGHEST POINT ON $a(t)$ (HORIZ. TANGENT)

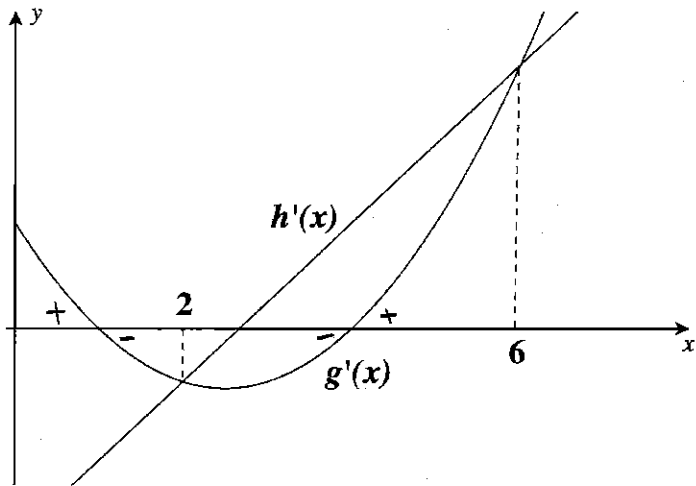
STEP 1 $a'(t) = -0.6t + 3.9$

STEP 2 Solve $a'(t) = 0$

HW 9.9/1:

Given $g'(x) = 2x^2 - 10x + 8$

$h'(x) = 6x - 16$



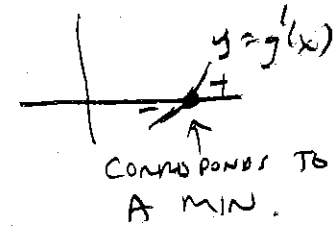
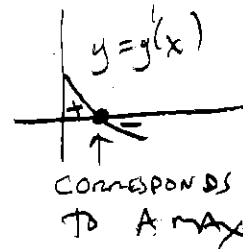
What does it mean when...

- (a) ... $g'(x)$ crosses the x-axis?
- ... $h'(x)$ crosses the x-axis?

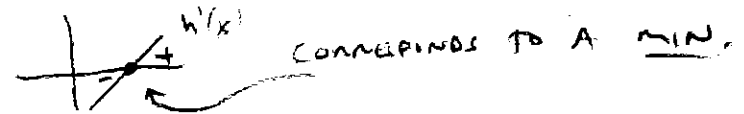
- (b) ... $g'(x)$ has a horizontal tangent
(and how do you find it)?

- (c) ... $h'(x)$ intersects $g'(x)$?

(a) $g'(x) \stackrel{?}{=} 0 \Leftrightarrow g(x)$ has a horizontal tangent



$h'(x) = 0 \Leftrightarrow h(x)$ has a horizontal tangent



(b) g' has a horiz. tangent $\Leftrightarrow g''(x) \stackrel{?}{=} 0$
 $4x - 10 \stackrel{?}{=} 0$



(c) $g''(x) = h''(x) \Leftrightarrow$ SAME SLOPE (RATE)
 $\Leftrightarrow g(x)$ AND $h(x)$ BIGGEST GAPS